

- 15.0** Demonstrate knowledge and proof of the fundamental theorem of calculus and use it to interpret integrals as antiderivatives.
- 16.0** Use definite integrals in problems involving area, velocity, acceleration, volume of a solid, area of a surface of revolution, length of a curve, and work.
- 17.0** Compute, by hand, the integrals of a wide variety of functions by using techniques of integration, such as substitution, integration by parts, and trigonometric substitution. They can also combine these techniques when appropriate.
- 18.0** Know the definitions and properties of inverse trigonometric functions and the expression of these functions as indefinite integrals.
- 19.0** Compute, by hand, the integrals of rational functions by combining the techniques in standard 17.0 with the algebraic techniques of partial fractions and completing the square.
- 20.0** Compute the integrals of trigonometric functions by using the techniques noted above.
- 21.0** Understand the algorithms involved in Simpson’s rule and Newton’s method. They use calculators or computers or both to approximate integrals numerically.
- 22.0** Understand improper integrals as limits of definite integrals.
- 23.0** Demonstrate an understanding of the definitions of convergence and divergence of sequences and series of real numbers. By using such tests as the comparison test, ratio test, and alternate series test, they can determine whether a series converges.
- 24.0** Understand and can compute the radius (interval) of the convergence of power series.

- 25.0** Differentiate and integrate the terms of a power series in order to form new series from known ones.
- 26.0** Calculate Taylor polynomials and Taylor series of basic functions, including the remainder term.
- 27.0** Know the techniques of solution of selected elementary differential equations and their applications to a wide variety of situations, including growth-and-decay problems.

**Senate Bill 2X  
High School Exit Exam Highlights**

- Senate Bill 2X requires all students completing grade twelve to pass a high school exit exam in language arts and math commencing in 2003–04.
- The bill requires the State Superintendent of Public Instruction to develop and the State Board of Education to approve the exam by October 1, 2000.
- Beginning in 2000–01, grade nine students will be eligible to take the exam.
- Beginning in 2001–02, grade ten students will be required to take the exam.
- The law does not make the exam a requirement for graduation until 2003–04.
- If a pupil does not possess sufficient English language skills to be assessed by the exit exam, the district may defer the requirement that the student pass the exam “for a period of up to 24 calendar months of enrollment in the California public school system until the pupil has completed six months of instruction in reading, writing, and comprehension in the English language.”

**College Entrance Requirements**

Parents generally know that many colleges require good high school grades for admission. Although grades are important, students do not have to have top grades to get into college. There are colleges for every student. You should also know that students need to take a specific series of college preparatory classes in high school, and the minimum requirements vary depending on the selected college or university. The a–g requirements noted below are submitted by the Regents of the University of California and are generally the most rigorous:

- a. An English class every semester of every year for four years.
- b. A mathematics class every semester of every year for three years, including algebra and geometry. Four years are recommended.
- c. Two years of a laboratory science beyond the ninth grade. An additional year is recommended.
- d. Two years of history–social science, which are to include U.S. government, world history, culture, and geography.
- e. Two years of the same language other than English.
- f. Two years of college preparatory electives in addition to those required in “a–e” above.
- g. One year of visual and performing arts, effective for the entering class of 2003.

Every high school has a list of acceptable classes and can tell you how many should be taken. At least one class in the area of visual or performing arts is a good choice for many students.

To gain admission to college, your children must also take either the Scholastic Assessment Test (SAT) or the American College Test (ACT) and submit the scores. Find out when the tests are given and be sure your children sign up to take one of them.

CALCULUS

*The  
California  
Mathematics  
Content  
Standards*

CALIFORNIA  
DEPARTMENT  
OF EDUCATION  
2001

# The California Mathematics Content Standards

ACADEMIC CONTENT STANDARDS IN CRITICAL curriculum areas are an important part of educational reform in California.

This brochure provides an overview of the California Department of Education’s approach to meeting the academic needs of your child in the core curricular areas (math, science, history-social science, and language arts). A copy of the mathematics content standards adopted by the State Board of Education is included.

Well-communicated standards spell out what students learn in a specific subject. School districts must adopt the state standards or use them as a foundation for creating their own district standards. When a school district develops standards, they must be as rigorous and challenging as the state standards.



The standards for grades eight through twelve are organized differently from those for kindergarten through grade seven. In this section strands are not used for organizational purposes as they are in the elementary grades because the mathematics studied in grades eight through twelve falls naturally under discipline headings: algebra, geometry, and so forth. Many schools teach this material in traditional courses; others teach it in an integrated fashion.

To allow local educational agencies and teachers flexibility in teaching the material, the standards for grades eight through twelve do not mandate that a particular discipline be initiated and completed in a single grade. The core content of these subjects must be covered;

students are expected to achieve the standards however these subjects are sequenced.

Standards are provided for algebra I, geometry, algebra II, trigonometry, mathematical analysis, linear algebra, probability and statistics, Advanced Placement probability and statistics, and calculus. Many of the more advanced subjects are not taught in every middle school or high school. Moreover, schools and districts have different ways of combining the subject matter in these various disciplines. For example, many schools combine some trigonometry, mathematical analysis, and linear algebra to form a precalculus course. Some districts prefer offering trigonometry content with algebra II.

What is described in this section are standards for the academic content by discipline; the document does not endorse a particular choice of structure for courses or a particular method of teaching the mathematical content.

## Calculus

When taught in high school, calculus should be presented with the same level of depth and rigor as are entry-level college and university calculus courses. These standards outline a complete college curriculum in one variable calculus. Many high school programs may have insufficient time to cover all of the following content in a typical academic year. For example, some districts may treat differential equations lightly and spend substantial time on infinite sequences and series. Others may do the opposite. Consideration of the College Board syllabi for the Calculus AB and Calculus BC sections of the Advanced Placement Examination in Mathematics may be helpful in making curricular decisions. Calculus is a widely applied area of mathematics and involves a beautiful intrinsic

theory. Students mastering this content will be exposed to both aspects of the subject.

### Students:

- 1.0** Demonstrate knowledge of both the formal definition and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, infinite limits, and limits at infinity. Students know the definition of convergence and divergence of a function as the domain variable approaches either a number or infinity:
  - 1.1 Prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions.
  - 1.2 Use graphical calculators to verify and estimate limits.
  - 1.3 Prove and use special limits, such as the limits of  $(\sin(x))/x$  and  $(1-\cos(x))/x$  as  $x$  tends to 0.
- 2.0** Demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function.
- 3.0** Demonstrate an understanding and the application of the intermediate value theorem and the extreme value theorem.
- 4.0** Demonstrate an understanding of the formal definition of the derivative of a function at a point and the notion of differentiability:
  - 4.1 Demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function.
  - 4.2 Demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change. Students can use derivatives to solve a variety of problems from physics, chemistry, economics, and so forth that involve the rate of change of a function.

- 4.3 Understand the relation between differentiability and continuity.
- 4.4 Derive derivative formulas and use them to find the derivatives of algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions.
- 5.0** Know the chain rule and its proof and applications to the calculation of the derivative of a variety of composite functions.
- 6.0** Find the derivatives of parametrically defined functions and use implicit differentiation in a wide variety of problems in physics, chemistry, economics, and so forth.
- 7.0** Compute derivatives of higher orders.
- 8.0** Know and can apply Rolle’s theorem, the mean value theorem, and L’Hôpital’s rule.
- 9.0** Use differentiation to sketch, by hand, graphs of functions. They can identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing.
- 10.0** Know Newton’s method for approximating the zeros of a function.
- 11.0** Use differentiation to solve optimization (maximum-minimum problems) in a variety of pure and applied contexts.
- 12.0** Use differentiation to solve related rate problems in a variety of pure and applied contexts.
- 13.0** Know the definition of the definite integral by using Riemann sums. They use this definition to approximate integrals.
- 14.0** Apply the definition of the integral to model problems in physics, economics, and so forth, obtaining results in terms of integrals.